Fluid Mechanics I

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Lecture 11

(Chapter 5: Work and Energy of Moving Fluids 1)

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Chapter Objectives

- To develop Euler's equations of motion and the Bernoulli equation for streamline coordinates, and to Illustrate some important applications.
- To show how to establish the energy grade line and hydraulic grade line for a fluid system.
- To develop the energy equation from the first law of thermodynamics, and to show how to solve problems that include pumps, turbines, and frictional loss.

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Euler's Equation of Motion

- Figure 5.14 shows a short section of a streamtube surrounding the streamline.
- Cross-sectional area is small enough for the velocity to be considered constant over the cross-section.
- AB and CD are two cross-sections separated by a short distance δs.
- P_{side} will be exerted on the sides of the element by surrounding fluid.

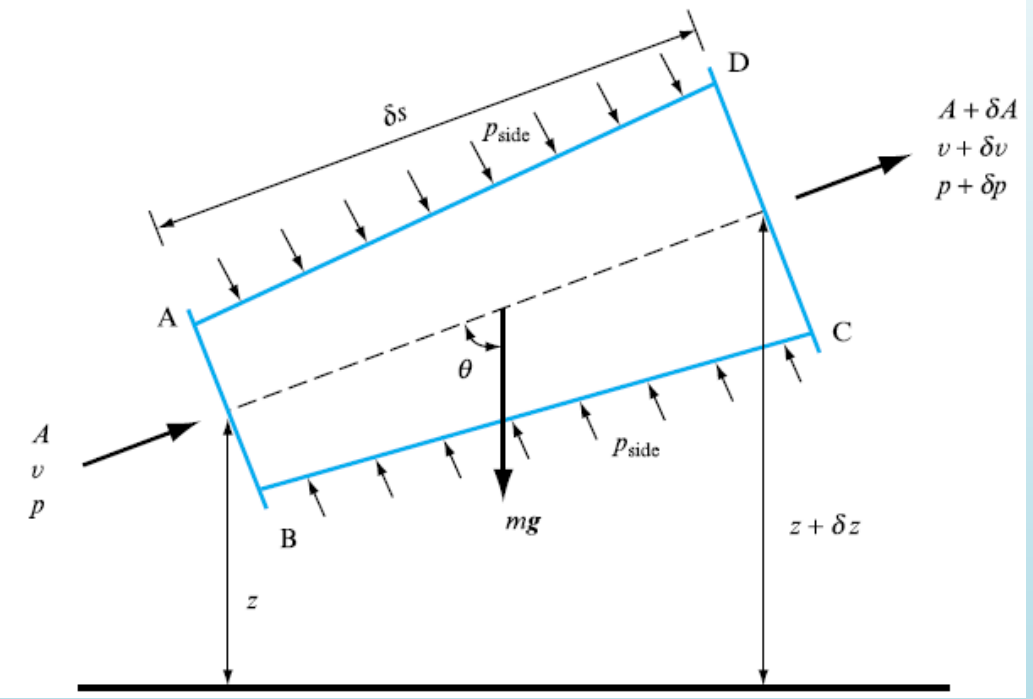


Fig. 5.14: Euler's Equation (Douglas et al., 2005)

- Weight of the element mg act vertically downward at an angle θ to the centreline.
- At AB the area is A, velocity v, pressure p and elevation z, while at CD the corresponding values are $A + \delta A$, $v + \delta v$, $p + \delta p$ and $z + \delta z$.

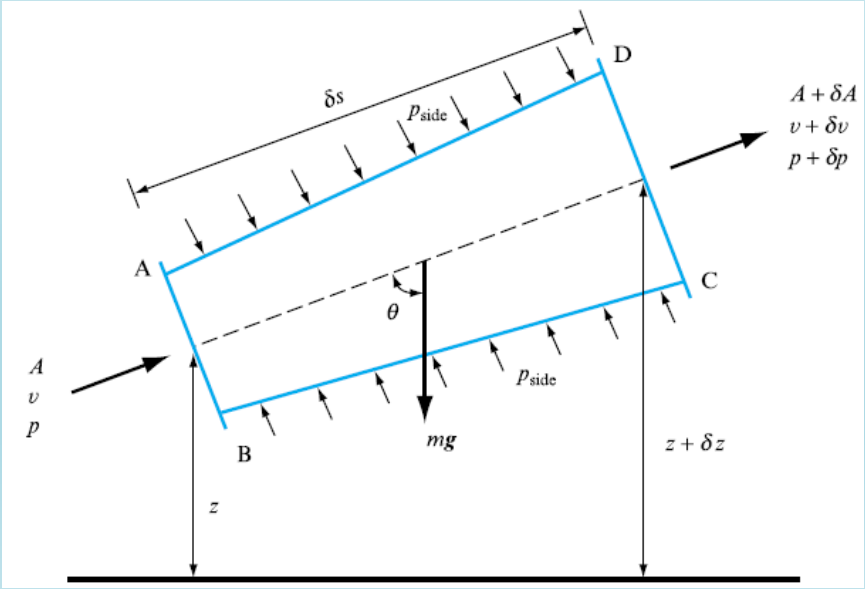
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Euler's Equation of Motion (Cont.)

- Mass per unit time flowing = $\rho A v$
- Rate of increase of momentum from AB to CD = $\rho A v\,[(v + \delta v) - v] = \rho A v \delta v$ ... (I)

Forces acting to produce this increase of momentum in the direction of motion are:

- Force due to p in direction of motion = pA
- Force due to $p + \delta p$ opposing motion = $(p + \delta p)(A + \delta A)$
- Force due to P_{side} producing a component in the direction of motion = $P_{side}\,\delta A$
- Force due to mg producing a component opposing motion = $mg\ Cos\ \theta$



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Euler's Equation of Motion (Cont.)

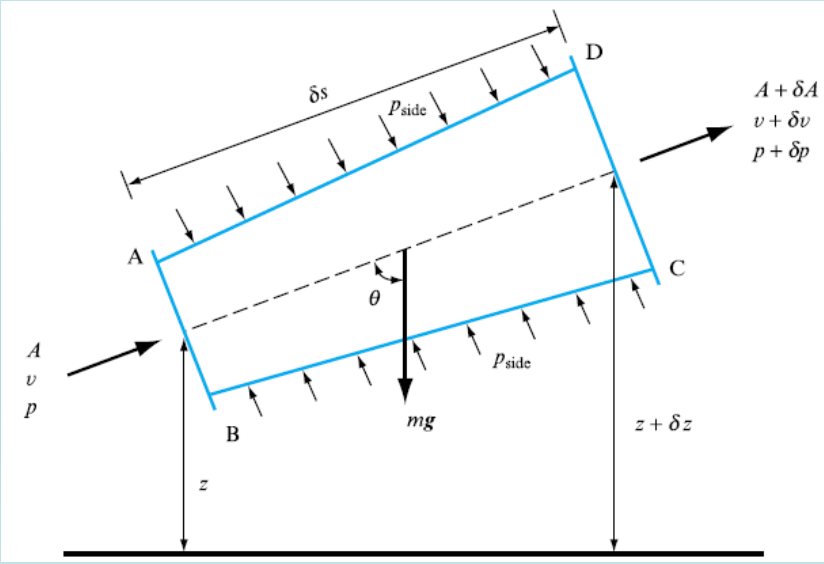
- Resultant force in the direction of motion =

$$pA - (p + \delta p)(A + \delta A) + P_{side}\,\delta A - mg\,Cos\,\theta$$

The value of P_{side} will vary from p at AB to $p + \delta p$ at CD and can be taken as $p + k\delta p$, where k is a fraction.

- Weight of element, $mg = \rho g * volume = \rho g \left(A + \frac{1}{2}\delta A\right)\delta s;\ \ Cos\,\theta = \frac{\delta z}{\delta s}$
- Resultant force in the direction of motion =

$$-p\,\delta A - A\,\delta p - \delta p\,\delta A + p\,\delta A + K\,\delta p.\delta A - \rho g \left(A + \frac{1}{2}\delta A\right)\delta s.\ \left(\frac{\delta z}{\delta s}\right)$$



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Euler's Equation of Motion (Cont.)

Neglecting products of small quantities, resultant force in the direction of motion =

$$-A\delta p - \rho g A \delta z \qquad \ldots\ldots\ldots (II)$$

Applying Newton's second law from equations (I) and (II):

$$\rho A v \delta v = -A\delta p - \rho g A \delta z$$

Dividing by $\rho A \delta s$:

$$\frac{1}{\rho}\frac{\delta p}{\delta s} + v\,\frac{\delta v}{\delta s} + g\frac{\delta z}{\delta s} = 0$$

In the limit of $\delta s \rightarrow 0$: $\frac{1}{\rho}\frac{dp}{ds} + v\,\frac{dv}{ds} + g\frac{dz}{ds} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (III)$

Equation (III) is Euler's differential equation of motion. It yields the relationship between pressure *p*, velocity *v*, density ρ and elevation *z* along a streamline for steady flow.

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The Bernoulli's Equation

For an incompressible fluid, for which ρ is constant, integration of equation (III) along the streamline, with respect to *s*, gives:

$$\frac{p}{\rho} + \frac{v^2}{2} + gz = constant \quad \ldots\ldots\ldots\ldots (IV)$$

- In equation IV, the terms represent energy per unit mass.

Dividing by g:

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = constant = H \quad \ldots\ldots\ldots\ldots (V)$$

- In equation V, the terms represent energy per unit weight.

Equation (V) is known as Bernoulli's equations. It yields the relationship between pressure *p*, velocity *v*, and elevation *z* for steady flow of a frictionless fluid of constant density.

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Application and Limitation of Bernoulli's Equation (Cont.)

Application of Bernoulli equation: It can be used to determine either the velocity or the pressure at different points on a streamline.

Limitation of Bernoulli equation: It can only be applied for steady flow of an ideal fluid.

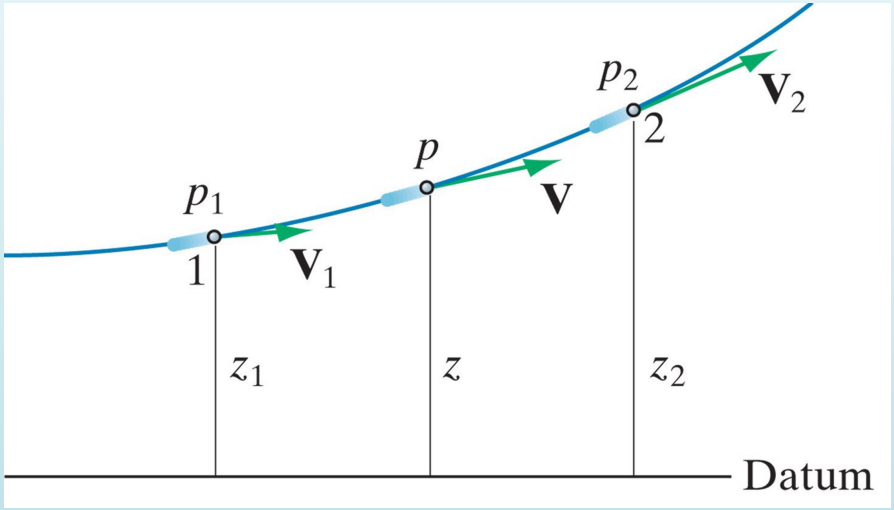


Fig. 5.4: Fluid particle travel between point 1 and 2 following streamline

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Applications of Bernoulli Equation

Flow from a Large Reservoir:

Assumption:

- Flow through the drain is steady.
- Elevation difference between C and D is small so that $V_C \approx V_B \approx V_D$
- Pressure at the centreline B of the opening is atmospheric, as it is at C and D.
- Fluid in the tank/reservoir is ideal fluid.
- Gravitational datum is at B and $P_A = P_B = 0$

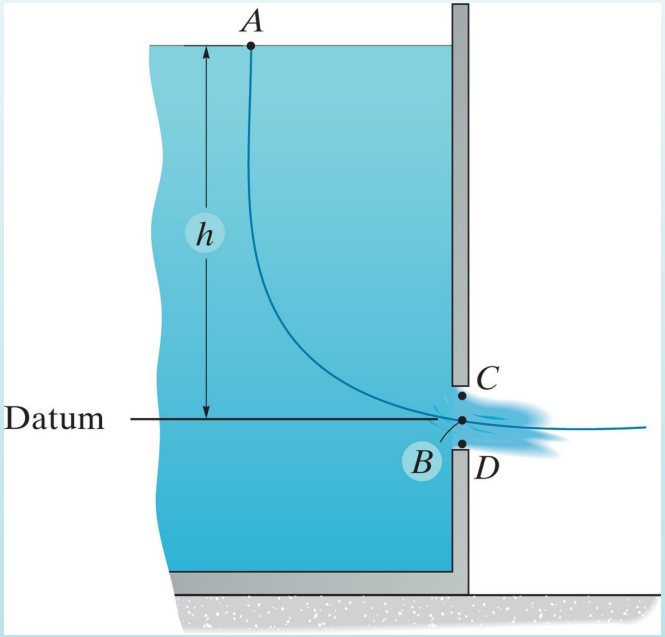


Fig. 5.7: Water flowing from a tank or reservoir through a drain

Bernoulli equation between points A and B lying on the streamline,

$$\frac{P_A}{\rho} + \frac{V_A^2}{2} + gZ_A = \frac{P_B}{\rho} + \frac{V_B^2}{2} + gZ_B$$

$$0 + 0 + gh = 0 + \frac{V_B^2}{2} + 0$$

$$V_B = \sqrt{2gh}$$

This result is known as Torricelli's law.

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Applications of Bernoulli Equation (Cont.)

Flow around a Curved Boundary:

- Fluid flows around a smooth obstacle.
- B is a stagnation point.
- Fluid particles moving from A to B must decelerate as they approach the boundary.
- Velocity of fluid particles at B momentarily becomes zero.

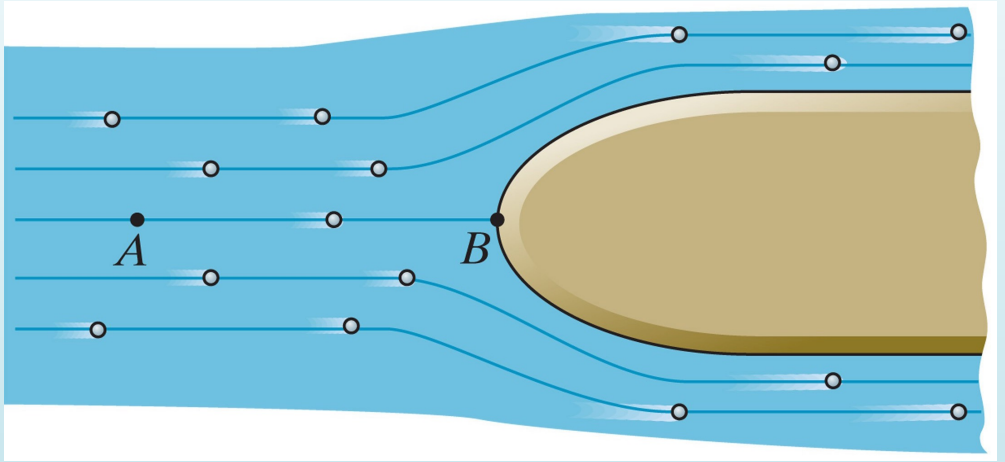


Fig. 5.8: fluid flow around a smooth obstacle

Bernoulli equation between points A and B,

$$\frac{P_A}{\rho} + \frac{V_A^2}{2} + gZ_A = \frac{P_B}{\rho} + \frac{V_B^2}{2} + gZ_B$$

$$\frac{P_A}{\rho} + \frac{V_A^2}{2} + 0 = \frac{P_B}{\rho} + 0 + 0$$

$$P_B = P_A + \rho\frac{V_A^2}{2}$$

Stagnation pressure | Static pressure | Dynamic pressure

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Applications of Bernoulli Equation (Cont.)

Flow in an Open Channel

- Immerse a bent tube called stagnation tube or a Pitot tube into the stream and observe the height h to which the liquid rises within the tube.
- Velocity and pressure at point A are V_A and P_A $(= \rho g d)$, respectively.
- Point B is stagnation point (velocity $V_B = 0$) and pressure at point B is $P_B = \rho g\,(d + h)$.

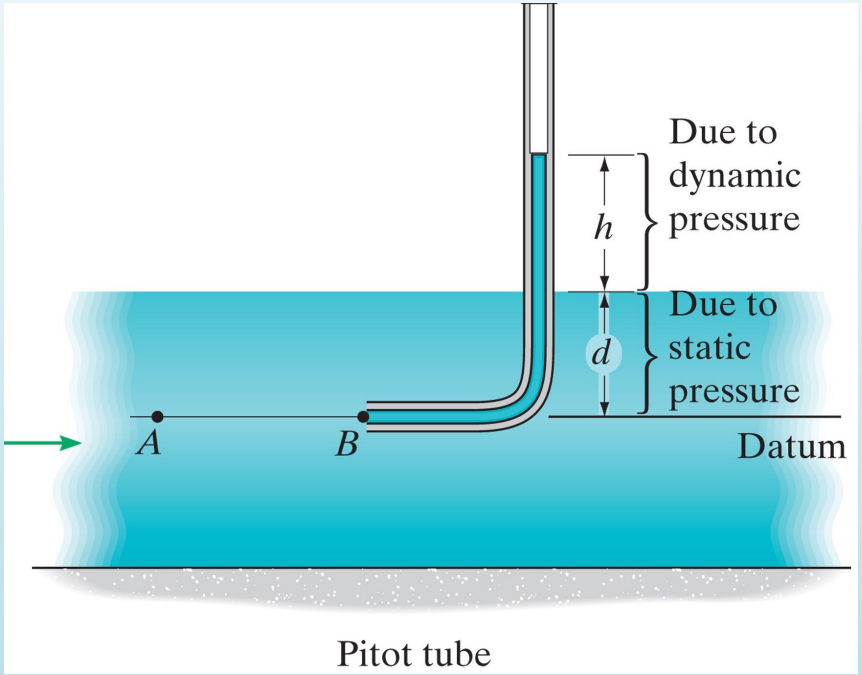


Fig. 5.9: fluid flow in an open channel

Applying the Bernoulli equation we have, $V_A = \sqrt{2gh}$

Thus, velocity of the flow can be determined by measuring h on the Pitot tube.

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Applications of Bernoulli Equation (Cont.)

Flow in a Closed Conduit

- Immerse a piezometer and a Pitot tube into the flow.
- Velocity and pressure at point A are V_A and $P_A = \rho g\,(h + d)$, respectively.
- Point B is stagnation point (velocity $V_B = 0$), and pressure at point B is $P_B = \rho g(h + d + l)$.

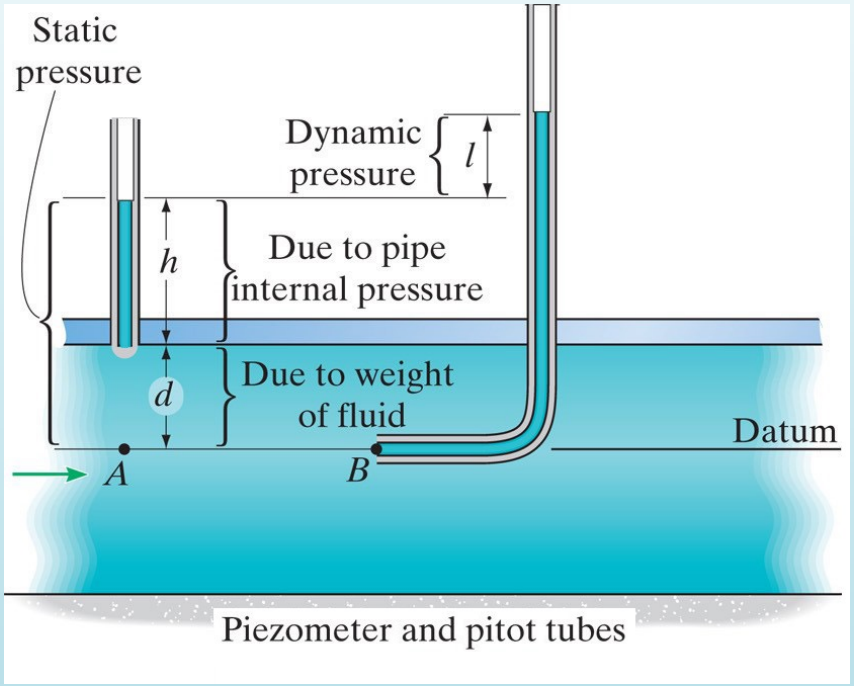


Fig. 5.10 (a): Fluid flow in a Closed Conduit

Applying the Bernoulli equation we have, $V_A = \sqrt{2gl}$

Thus, velocity of the flow can be determined by measuring l on the Pitot tube.

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Applications of Bernoulli Equation (Cont.)

Flow in a Closed Conduit (Cont.)

Velocity of the flow can also be determined by Immersing a single tube called Pitot-static tube.

- Elevation difference between C and E is neglected.
- Pressure at A can be measured from the pressure tap at C ($P_A = P_C + \rho gh$).
- stagnation pressure at B can be measured from the pressure tap at E ($P_B = P_E + \rho gh$).

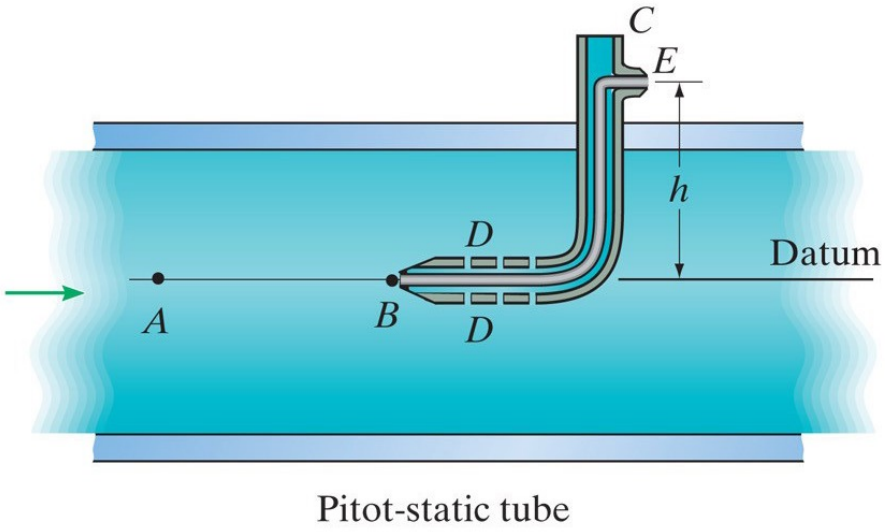


Fig. 1.10 (b): Fluid flow in a Closed Conduit

Applying the Bernoulli equation we have,

$$V_A = \sqrt{\frac{2}{\rho}(P_E - PC)}$$

(P_E - P_C) can be determined by attaching a manometer to outlets C and E.

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Applications of Bernoulli Equation (Cont.)

Venturi Meter

- It is used to measure the average velocity or the flow of an incompressible fluid through a pipe.

- Applying the Bernoulli equation between points 1 and 2 results,

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gZ_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gZ_2$$

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + 0 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + 0$$

- Applying continuity equation at points 1 and 2 results,

$$\frac{\partial}{\partial t}\int_{CV} \rho \, dV + \int_{CS} \rho V \, dA = 0$$

$$0 - V_1 \pi \frac{d_1^2}{4} + V_2 \pi \frac{d_2^2}{4} = 0$$

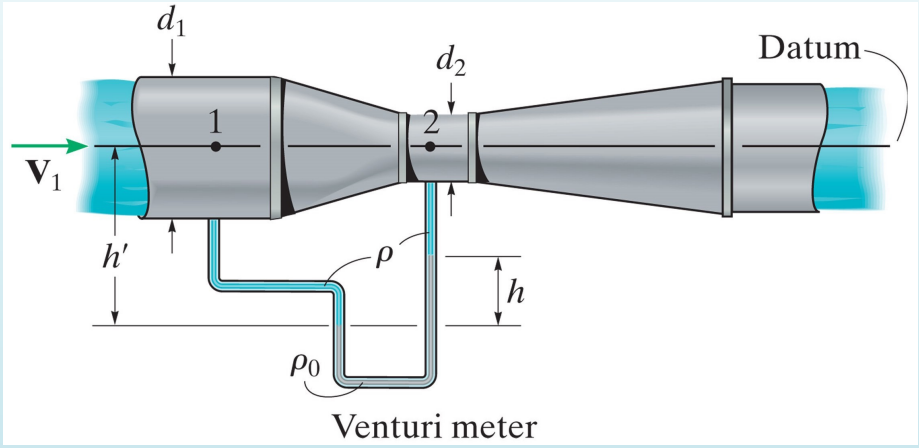


Fig. 5.11: Venturi meter

- Combining results from Bernoulli equation and Continuity equation for V_1 gives,

$$V_1 = \sqrt{\frac{2\,(P_2 - P_1)/\rho}{1 - (d_1/d_2)^4}}$$

$P_1 - P_2$ is measured using a pressure transducer or a manometer.

Volumetric flow can be measures from the equation: $Q = V_1 A_1$

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Example 5.3

Example: 5.3 (Page 237)

EXAMPLE 5.3

Determine the average velocity of the flow of water in the pipe in Fig. 5–13, and the static and dynamic pressure at point B. The water level in each of the tubes is indicated. Take $\rho_w = 1000\,\text{kg/m}^3$.

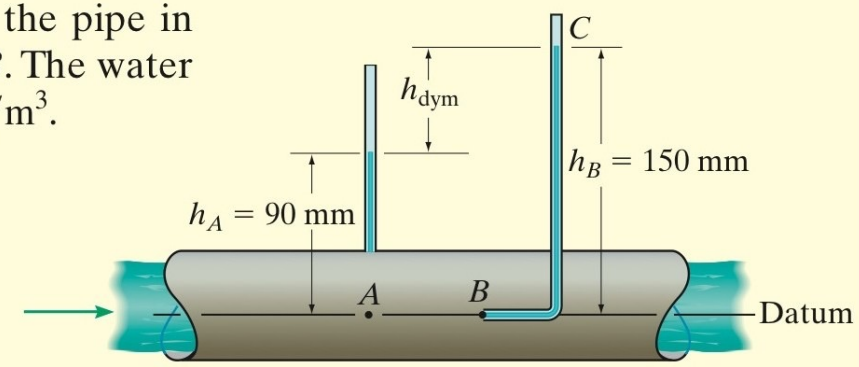


Fig. 5–13

Given:

Water level in each tubes are given.

Density of water = 1000 Kg/m^3

Required:

Average velocity of the flow of water in the pipe = ?

Static and dynamic pressure at point B = ?

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Example 5.3 (Cont.)

EXAMPLE 5.3 CONTINUED

Fluid Description. We have steady flow. Also, we will consider the water to be an ideal fluid.

Bernoulli Equation. At A the total pressure is the *static pressure* found from $p_A = \rho_w g h_A$, and at B the total (or stagnation) pressure is a *combination* of static and dynamic pressures found from $p_B = \rho_w g h_B$. Knowing these pressures, we can determine the average velocity of flow V_A using the Bernoulli equation, applied at points A and B, where B is a stagnation point on the streamline. We have

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + g z_A = \frac{p_B}{\rho} + \frac{V_B^2}{2} + g z_B$$

$$\frac{\rho_w g h_A}{\rho_w} + \frac{V_A^2}{2} + 0 = \frac{\rho_w g h_B}{\rho_w} + 0 + 0$$

$$\frac{V_A^2}{2} = g(h_B - h_A) = \left(9.81\,\text{m/s}^2\right)(0.150\,\text{m} - 0.090\,\text{m})$$

$$V_A = 1.085\ \text{m/s} = 1.08\ \text{m/s}$$

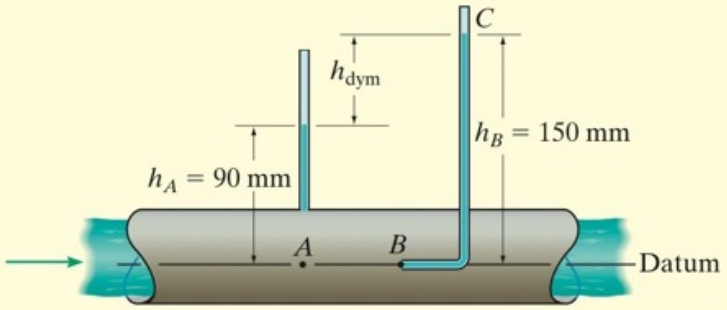


Fig. 5–13

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Example 5.3 (Cont.)

EXAMPLE 5.3 CONTINUED

The static pressure at both A and B is determined from the piezometric head.

$$(p_A)_{\text{static}} = (p_B)_{\text{static}} = \rho_w g h_A = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^3\right)(0.09\ \text{m}) = 883\,\text{Pa} \qquad \textit{Ans.}$$

The dynamic pressure at B is determined from

$$\rho_w \frac{V_A^2}{2} = \left(1000\ \text{kg/m}^3\right)\frac{(1.085\ \text{m/s})^2}{2} = 589\,\text{Pa} \qquad \textit{Ans.}$$

This value can also be obtained by noticing that

$$h_{\text{dyn}} = 0.15\ \text{m} - 0.09\ \text{m} = 0.06\ \text{m}$$

so that

$$(p_B)_{\text{dyn}} = \rho_w g h_{\text{dyn}} = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.06\ \text{m}) = 589\,\text{Pa} \qquad \textit{Ans.}$$

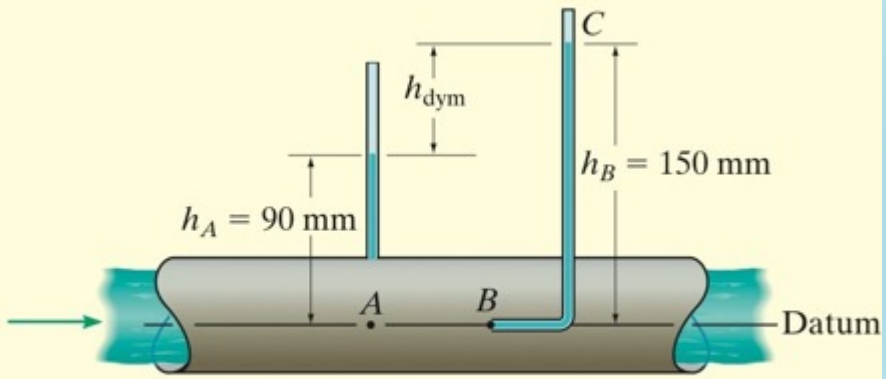


Fig. 5–13

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Example 5.5

Example: 5.5 (Page 239)

EXAMPLE 5.5

Water flows up through the vertical pipe that is connected to the transition, Fig. 5–15. If the volumetric flow is 0.02 m^3/s, determine the height h to which the water will rise in the Pitot tube. The piezometer level at A is indicated.

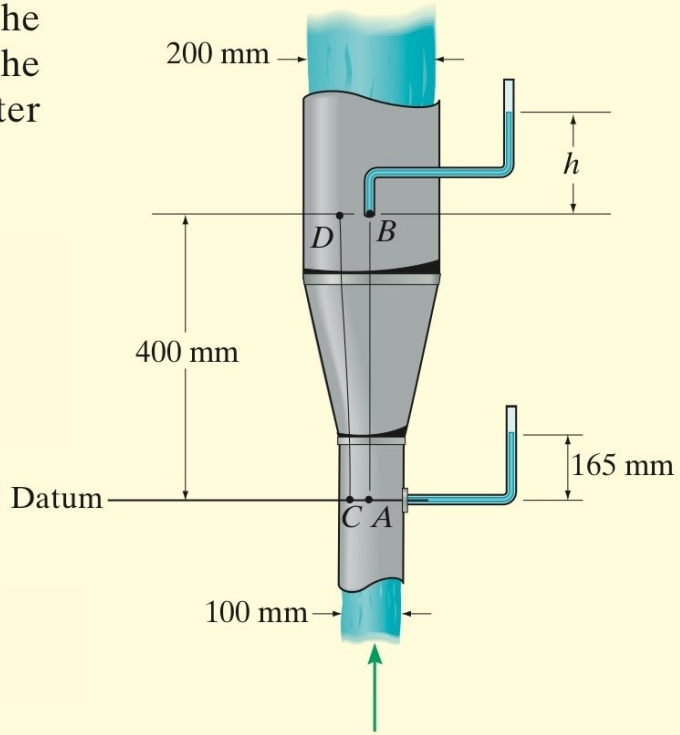


Fig. 5–15

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Example 5.5 (Cont.)

Given:
Volumetric flow of water Q = 0.02 m^3/s
Piezometer level at A is given.

Required:
Height h to which the water will rise in the Pitot tube = ?

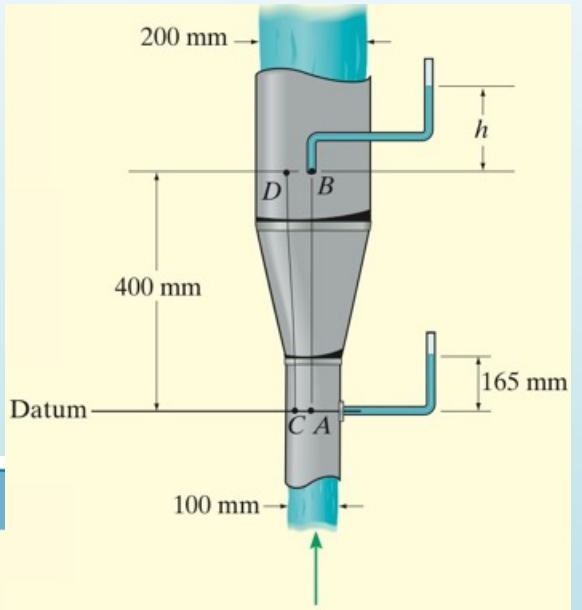


Fig. 5–15

EXAMPLE 5.5 CONTINUED

SOLUTION

Fluid Description. The flow is steady, and the water is assumed to be an ideal fluid, where $\rho_w = 1000\ kg/m^3$.

Bernoulli Equation. From the piezometer reading, the pressure at A is

$$p_A = \rho_w g h_A = (1000\ kg/m^3)(9.81\ m/s^2)(0.165\ m) = 1618.65\ Pa$$

This total pressure is caused by the static pressure in the water. In other words, it is the pressure within the enclosed pipe at this level.

Since the flow is known, the velocity at A can be determined.

$$Q = V_A A_A; \qquad 0.02\ m^3/s = V_A[\pi(0.05\ m)^2]$$

$$V_A = 2.546\ m/s$$

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Example 5.5 (Cont.)

EXAMPLE 5.5 CONTINUED

Also, since B is a stagnation point, $V_B = 0$. We can now determine the pressure at B by applying the Bernoulli equation at points A and B on the vertical streamline in Fig. 5–15. The datum is placed at A, and so

$$\frac{p_A}{\rho_w} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho_w} + \frac{V_B^2}{2} + gz_B$$

$$\frac{1618.65\ \text{N/m}^2}{1000\ \text{kg/m}^3} + \frac{(2.546\ \text{m/s})^2}{2} + 0 = \frac{p_B}{1000\ \text{kg/m}^3} + 0 + (9.81\ \text{m/s}^2)(0.4\ \text{m})$$

$$p_B = 936.93\ \text{Pa}$$

Since B is a stagnation point, this total pressure is caused by *both* static and dynamic pressures at B. For the Pitot tube we require

$$h = \frac{p_B}{\rho_w g} = \frac{936.93\ \text{Pa}}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} = 0.09551\ \text{m} = 95.5\ \text{mm} \quad \textit{Ans.}$$

NOTE: Although it is not part of this problem, the pressure at D can be obtained ($p_D = 734$ Pa) by applying the Bernoulli equation along the streamline CD. First, though, the velocity $V_D = 0.6366$ m/s must be obtained by applying $Q = V_D A_D$.

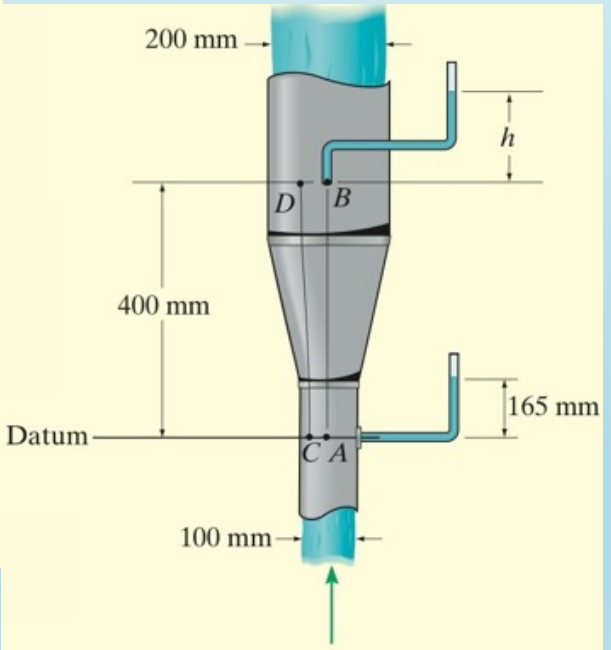


Fig. 5–15

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Reference

1. Fluid Mechanics, R.C. Hibbeler, Pearson Education Canada.

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Question?

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