Fluid Mechanics I

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Lecture 18

(Chapter 7: Differential Fluid Flow 2)

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The Euler and Bernoulli Equations

For ideal fluid (no viscous shear stress on the fluid particle), equations of motion are defined as Euler equations of motion. These equations are:

$$\rho g_x - \frac{\partial p}{\partial x} = \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)$$

$$\rho g_y - \frac{\partial p}{\partial y} = \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) \ldots\ldots\ldots\ldots (7.18)$$

$$\rho g_z - \frac{\partial p}{\partial z} = \rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right)$$

For steady two dimensional flow, Euler equations of motion become:

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \quad \ldots\ldots\ldots\ldots\ldots (7.20)$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial y} - g = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} \quad \ldots\ldots\ldots\ldots (7.21)$$

For ideal fluid, if the flow is irrotational and steady then the Bernoulli equation can be applied between any two points (x_1, y_1) and (x_2, y_2) that are not necessarily on the same streamline and we have,

$$\frac{P}{\rho} + \frac{V^2}{2} + gy = Const. \quad \ldots\ldots\ldots\ldots\ldots (7.24)$$

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Example 7.3

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EXAMPLE 7.3

The velocity field $\mathbf{V} = \{-6x\mathbf{i} + 6y\mathbf{j}\}$ m/s defines the two-dimensional ideal fluid flow in the vertical plane shown in Fig. 7–14. Determine the volumetric dilatation rate and the rotation of a fluid element located at point $B(1\text{ m}, 2\text{ m})$. If the pressure at point $A(1\text{ m}, 1\text{ m})$ is 250 kPa, what is the pressure at point B? Take $\rho = 1200\text{ kg/m}^3$.

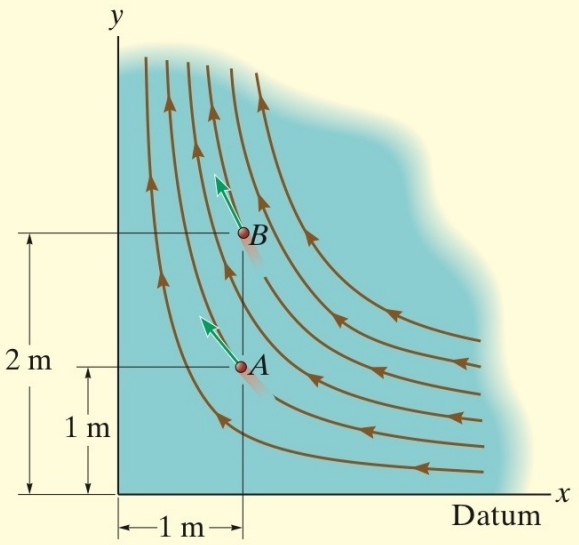


Fig. 7–14

Given:

Pressure at point A = 250 Kpa; Density ρ = 1200 Kg/ m^3.

Required:

- Volumetric dilation rate and the rotation of fluid element located at point B.
- Pressure at point B.

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Example 7.3 (Cont.)

EXAMPLE 7.3 CONTINUED

SOLUTION

Fluid Description. Since the velocity is not a function of time, the flow is steady. The fluid is an ideal fluid.

Volumetric Dilatation. Applying Eq. 7–1, where $u = (-6x)\ \text{m/s}$, $v = (6y)\ \text{m/s}$, and $w = 0$, we have

$$\frac{\delta \forall / d\forall}{\partial t} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = -6 + 6 + 0 = 0 \qquad \textit{Ans.}$$

The result confirms that there is no change in the volume of the fluid element at B as it displaces.

Rotation. The angular velocity of the fluid element at B is defined by Eq. 7–2.

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(0 - 0) = 0 \qquad \textit{Ans.}$$

Therefore, the fluid element will not rotate about the z axis. Actually, the above two results apply at *all points* in the fluid, since they are independent of x and y. In other words, an ideal fluid is incompressible and produces irrotational flow.

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Example 7.3 (Cont.)

EXAMPLE 7.3 CONTINUED

Pressure. Since the flow is irrotational and steady, we can apply the Bernoulli equation at two points *not* located on the same streamline, Fig. 7–14. The velocities at A and B are

$$V_A = \sqrt{[-6(1)]^2 + [6(1)]^2} = 8.485 \text{ m/s}$$

$$V_B = \sqrt{[-6(1)]^2 + [6(2)]^2} = 13.42 \text{ m/s}$$

Therefore, with the datum at the x axis, we have

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + y_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + y_B$$

$$\frac{250(10^3)\text{ N/m}^2}{(1200\text{ kg/m}^3)(9.81\text{ m/s}^2)} + \frac{(8.485\text{ m/s})^2}{2(9.81\text{ m/s}^2)} + 1\text{ m} = \frac{p_B}{(1200\text{ kg/m}^3)(9.81\text{ m/s}^2)} + \frac{(13.42\text{ m/s})^2}{2(9.81\text{ m/s}^2)} + 2\text{ m}$$

$$p_B = 173\text{ kPa} \qquad \textit{Ans.}$$

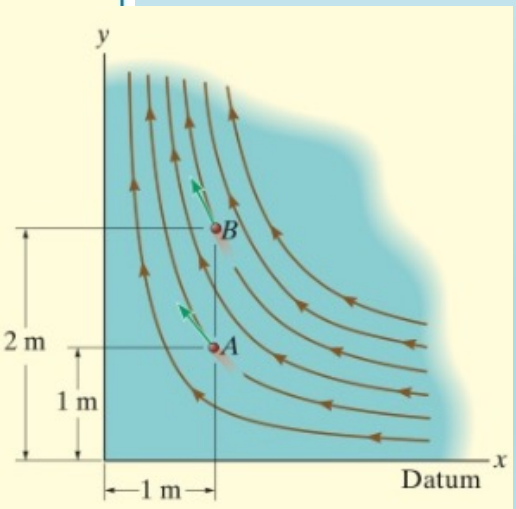


Fig. 7–14

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The Stream Function

- The Stream Function replaces two unknown velocity components **u** and **v** of equation of continuity in order to reduce the number of unknowns and to simplify the analysis of an ideal fluid flow problems.

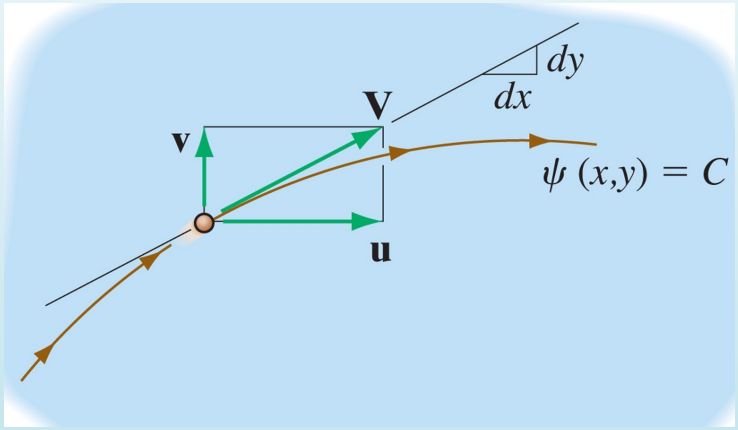


Fig. 7.15: Fluid streamline

- The stream function (Ψ) equation represents all the equations of the streamlines.

- Two components of velocity **u** and **v** can be related to Ψ as,

$$u = \frac{\partial \psi}{\partial y} \quad ; \qquad v = -\frac{\partial \psi}{\partial x} \ldots\ldots (7.27)$$

- For polar coordinate system (**r**, **θ**), velocity components can be related to Ψ as,

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \; ; \qquad v_\theta = -\frac{\partial \psi}{\partial r} \ldots\ldots (7.28)$$

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The Stream Function (Cont.)

- The Stream Function can also be used to determine the volumetric flow between any two streamlines.

- Volumetric flow **dq** for triangular differential control volume located within the streamtube between the two streamlines Ψ and $\Psi + d\Psi$ can be expressed as,

$$dq = d\Psi = (\Psi + d\Psi) - \Psi$$

- The volume flow rate between any two streamlines can be expressed as:

$$q = \int_{\Psi_1}^{\Psi_2} d\Psi = \Psi_2(x, y) - \Psi_1(x, y) = C_2 - C_1 \ldots\ldots\ldots\ldots (7.29)$$

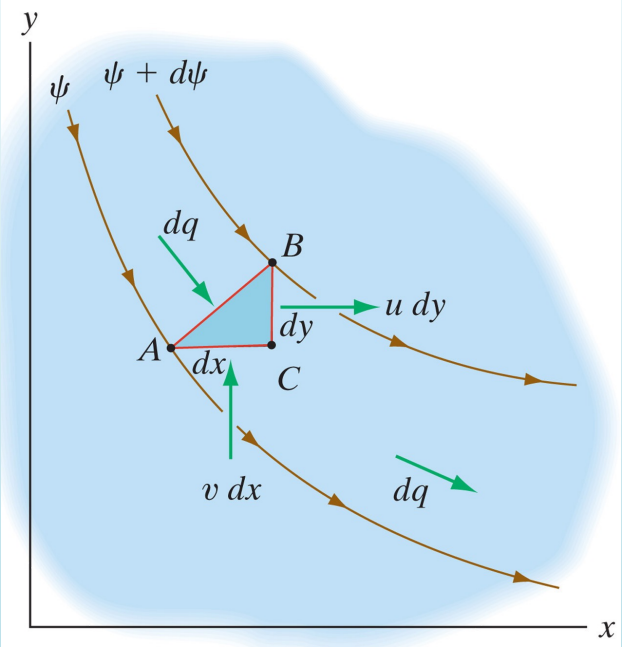
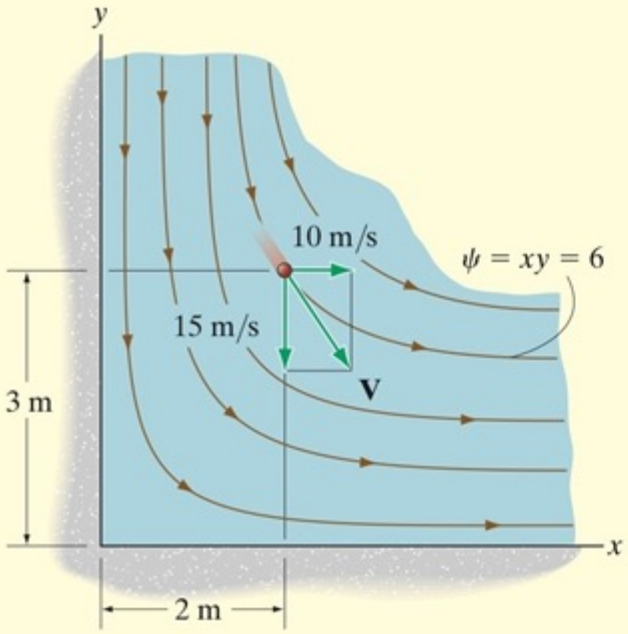


Fig. 7.17 (b): triangular differential control volume

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Example 7.6

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Flow around a 90° corner

Fig. 7–20

The streamlines for steady ideal fluid flow around the 90° corner in Fig. 7–20 are defined by the stream function $\psi(x, y) = (5xy)\ m^2/s$. Determine the velocity of the flow at point $x = 2$ m, $y = 3$ m. Can the Bernoulli equation be applied between any two points within this flow?

Required

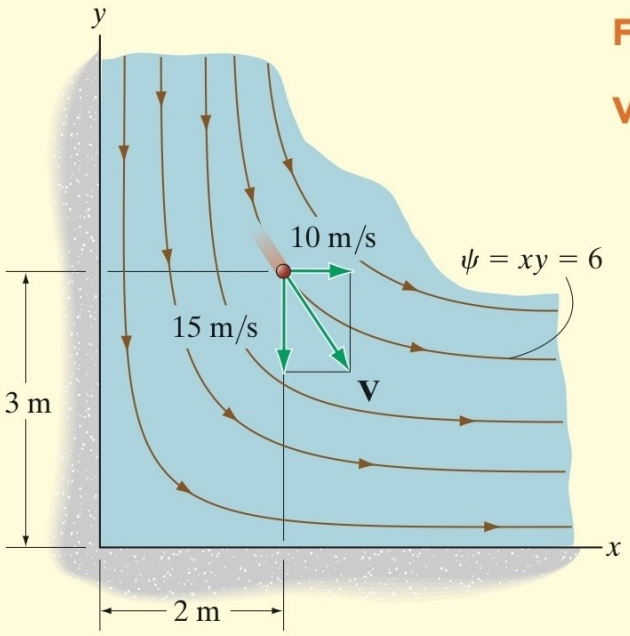
Velocity of the flow at point x = 2 m, y = 3 m.

Applicability of Bernoulli equation between any points within the flow.

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Example 7.6 (Cont.)

EXAMPLE 7.6 CONTINUED



Flow around a 90° corner

Fig. 7–20

SOLUTION

Fluid Description. As stated, we have steady ideal fluid flow.

Velocity. The velocity components are determined from Eqs. 7–27.

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial}{\partial y}(5xy) = (5x)\ \text{m/s}$$

$$v = -\frac{\partial \psi}{\partial x} = -\frac{\partial}{\partial x}(5xy) = (-5y)\ \text{m/s}$$

At point $x = 2\,\text{m}, y = 3\,\text{m}$,

$$u = 5(2) = 10\ \text{m/s}$$

$$v = -5(3) = -15\ \text{m/s}$$

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Example 7.6 (Cont.)

EXAMPLE 7.6 CONTINUED

The resultant velocity has a magnitude of

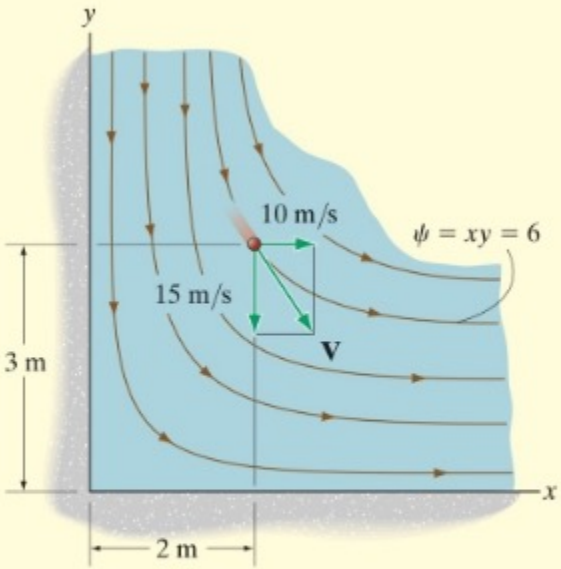
$$V = \sqrt{(10\ \text{m/s})^2 + (-15\ \text{m/s})^2} = 18.0\ \text{m/s} \qquad \textit{Ans.}$$

Its direction is tangent to the streamline that passes through point (2 m, 3 m), as shown in Fig. 7–20. To find the equation that defines this streamline, we require $\psi(x, y) = 5(2)(3) = C = 30\ \text{m}^2/\text{s}$. Thus $\psi(x, y) = 5xy = 30$, or $xy = 6$.

The Bernoulli equation can be applied since an ideal fluid has irrotational flow. Applying Eq. 7–2 to check this, we have

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}\left(\frac{\partial(-5y)}{\partial x} - \frac{\partial(5x)}{\partial y}\right) = 0$$

Therefore the Bernoulli equation can be used to find the pressure differences between any two points within the fluid.



Flow around a 90° corner

Fig. 7–20

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The Potential Function

The Potential Function (Φ) replaces two unknown velocity components u and v of equation of continuity in order to reduce the number of unknowns.

- Two components of velocity u and v can be related to Φ as,

$$u = \frac{\partial \Phi}{\partial x} \quad ; \qquad v = \frac{\partial \Phi}{\partial y} \qquad \ldots\ldots\ldots\ldots (7.30)$$

- For polar coordinate system (r, θ), velocity components can be related to Φ as,

$$v_r = \frac{\partial \Phi}{\partial r} \quad ; v_\theta = \frac{1}{r}\frac{\partial \Phi}{\partial \theta} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (7.32)$$

- Potential function $\Phi\,(x, y)$ only describes irrotational flow ($\omega = 0$).

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The Potential Function (Cont.)

- The velocity will always be perpendicular to any equipotential line $\Phi(x, y) = C'$ i.e. any equipotential line will be perpendicular to any intersecting streamline $\Psi(x, y) = C$.

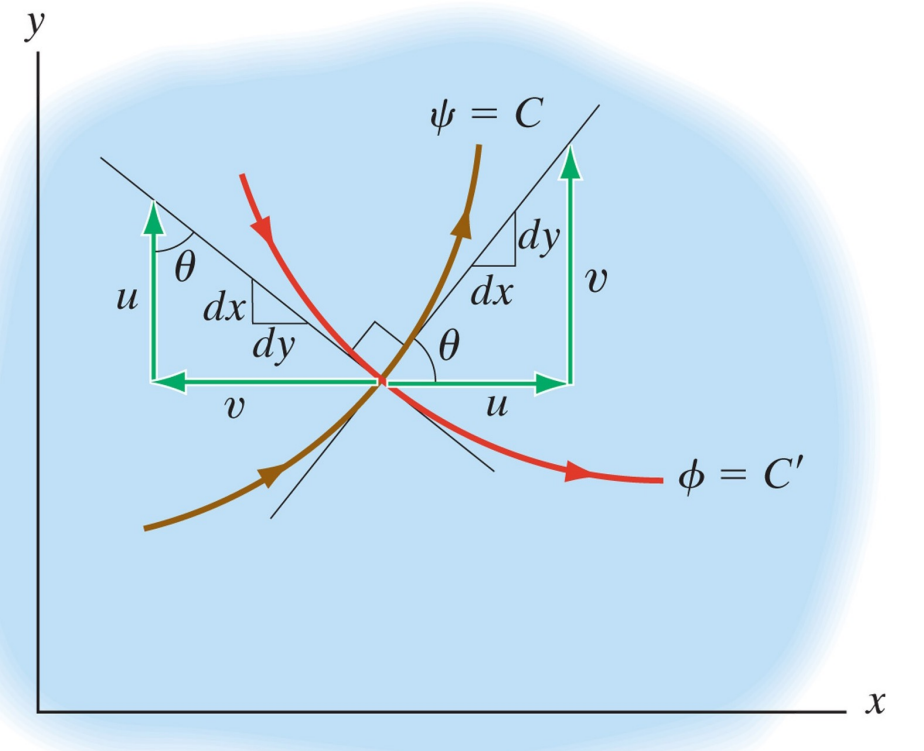


Fig. 7.21: Fluid flow showing position of velocity, potential and streamline.

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Flow Net

- A family of streamlines and equipotential lines for various values of the constants C and C' makes up a flow net.
- Flow net can serve as a graphical aid for visualizing the flow.

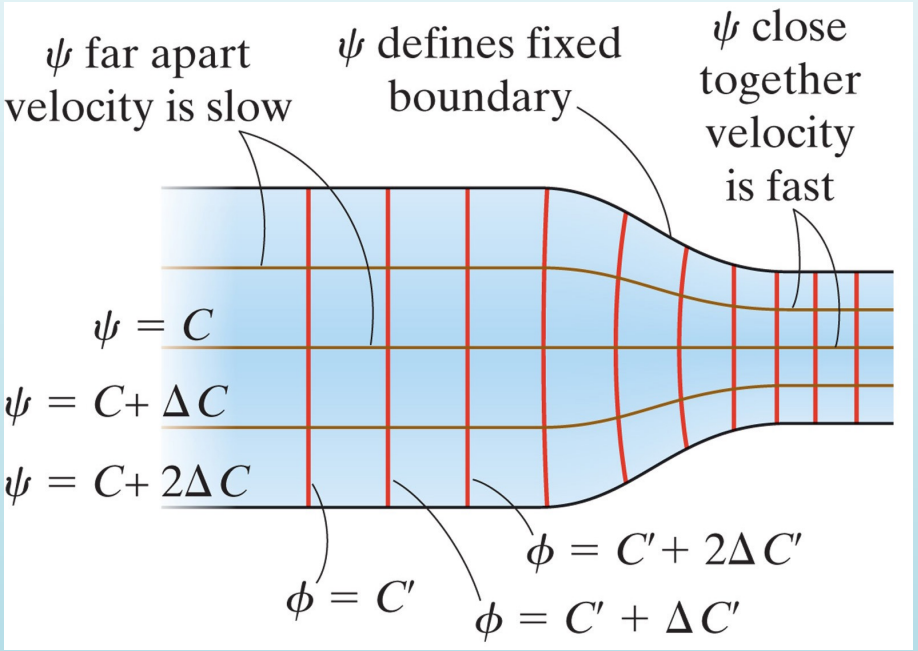


Fig. 7.22: Flow Net for ideal flow through a transition.

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Example 7.7

Example: 7.7 (Page 379)

A flow has a velocity field defined by $V = \{4xy^2\mathbf{i} + 4x^2y\mathbf{j}\}$ m/s. Is it possible to establish a potential function for this flow, and if so, what is the equipotential line passing through point $x = 1$ m, $y = 1$ m?

Given

Equation to define velocity profile.

Required

Possibility of establishing a potential function for the flow.

Equipotential line passing through point x= 1 m, y = 1 m.

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Example 7.7 (Cont.)

SOLUTION

Fluid Description. We have steady flow since V is not a function of time.

Analysis. A potential function can be developed *only if the flow is irrotational*. To find out if it is, we apply Eq. 7–2. Here $u = 4xy^2$ and $v = 4x^2y$, so that

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(8xy - 8xy) = 0$$

Therefore, integration gives

$$f(y) = C'$$

The potential function is then determined from Eq. 1, where, for convenience $C' = 0$, so that

$$\phi(x, y) = 2x^2y^2$$

To find the equipotential line passing through point (1 m, 1 m), we require $\phi(x, y) = 2(1)^2(1)^2 = 2$. Thus, $2x^2y^2 = 2$ or

$$xy = 1 \qquad \textit{Ans.}$$

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The Navier-Stokes Equations

- In the early 19th century the French engineer Louis Navier, and several years later the British mathematician George Stokes developed equations of motion while incorporating the viscosity in addition to gravity and pressure force.

- Navier-Stokes equations are:

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=\rho g_x-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=\rho g_y-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)\ldots\ldots\ldots..(7.75)$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=\rho g_z-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$$

- Navier-Stokes equations are applied to uniform, non-uniform, steady, or non-steady flow of an incompressible Newtonian fluid, for which μ is constant.

- Navier-Stokes equations can also be developed for cylindrical coordinate system.

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Computational Fluid Dynamics (CFD)

CFD is Widely used to design and analyse many different types of fluid flow problems such as those involving aircraft, pumps and turbines, heating and ventilation equipment, wind loadings on buildings, chemical processes, and even biomedical implant devices, as well as atmospheric weather modeling.

Several popular CFD computer programs such as FLUENT, FLOW - 3D, and ANSYS are currently available.

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Computational Fluid Dynamics (Cont.)

There are three basic parts to any CFD code – the input, the program, and the output.

- The operator must input data related to the fluid properties, specify whether laminar or turbulent flow occurs and identify the geometry of the boundary for the flow.

- The CFD Program consists of two parts. The first considers the fluid as a system of discrete particles and converts the relevant partial differential equations into a group of algebraic equations, and the second uses an iterative procedure to find solutions of these equations that satisfy the initial and boundary conditions for the problem. Several approaches such as finite difference method, finite element method, and finite control volume method can be used to do this.

- The output is usually a graphical form of the flow domain, showing the geometry of the problem, and sometimes the grid or mesh used for the analysis.

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Question?

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Good Luck

and

Thanks for Being with my Couse

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