Fluid Mechanics I

Prof. Dr. M. Enamul Hossain

CEO & President
NSRIC Chair Professor in Sustainable Energy
Online Education Division (OED)
NSRIC Inc.
Toronto, ON, Canada
E-mail: enamulh@nsric.ca



Lecture 21

(Chapter 8: Dimensional Analysis and Similitude 2)

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Lecture Outline

- To discuss how flow behavior depends on the types of forces that influence the flow, and to present an important set of dimensionless numbers that involve these forces.

- To formalize a dimensional analysis procedure by obtaining groups of dimensionless numbers using the Buckingham Pi theorem.

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Important Dimensionless Numbers (cont'd)

- **Euler Number:**

  - ✓ Difference in static pressure between two points in a fluid causes the fluid to flow.
  - ✓ The ratio of the force caused by this pressure difference to the inertia force, $\rho v^2 L^2$, is called the Euler number or the pressure coefficient.
  - ✓ The pressure force ΔpA can be expressed in terms of its length dimensions as ΔpL^2.
  - ✓ So the Euler number becomes,

$$E_u = \frac{Pressure\ force}{Inertia\ force} = \frac{\Delta p}{\rho v^2} \ldots\ldots\ldots\ldots (8.1)$$

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Important Dimensionless Numbers (cont'd)

- **Euler Number (cont'd):**
  - ✓ It controls the flow behavior when pressure and inertia forces are dominant, as when a liquid flows through a pipe.
  - ✓ It also plays a role in the cavitation of a liquid, and in the study of drag and lift effects produced by a fluid.

- **Reynolds Number:**
  - ✓ The ratio of the inertia force $\rho v^2 L^2$ to the viscous force is called the *Reynolds number.*
  - ✓ The viscous force has length dimensions of $\mu(v/L)L^2$ *or* μvL
  - ✓ So, the Reynolds number becomes,

$$N_{Re} = \frac{Inertia\ force}{Viscous\ force} = \frac{\rho vL}{\mu} \ldots\ldots\ldots\ldots (8.2)$$

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Important Dimensionless Numbers (cont'd)

- **Reynolds Number (cont'd):**
  - ✓ It was first developed by the British engineer Osborne Reynolds while investigating the behavior of laminar and turbulent flow in pipes.
  - ✓ The "characteristic length" L was chosen as the diameter of the pipe.
  - ✓ When R_e is large, the inertia forces within the flow will be greater than the viscous forces, and when this occurs, the flow will become turbulent.
  - ✓ Using this idea, Reynolds was able to roughly predict when laminar flow begins to transition to turbulent flow.
  - ✓ The Reynolds number has important implications in a number of flow phenomena.

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Important Dimensionless Numbers (cont'd)

- **Froude Number:**
  - ✓ The square root of the ratio of the inertia force to the fluid's weight $(\rho g)L^3$ called the *Froude number.* $\rho v^2 L^2$

$$F_r = \sqrt{\frac{Inertia\ force}{Gravitational\ force}} = \frac{v}{\sqrt{gL}} \ldots\ldots\ldots\ldots (8.3)$$

  - ✓ This number is named in honor of William Froude.
  - ✓ Its value indicates the relative importance of inertia and gravity effects on fluid flow.
  - ✓ If the Froude number is greater than unity, then inertia effects will outweigh those of gravity.
  - ✓ This number is important in the study of any flow having a free surface, as in the case of open channels, or the flow over dams or spillways.

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Important Dimensionless Numbers (cont'd)

- **Weber Number:**

  - ✓ The ratio of the inertia force $\rho v^2 L^2$ to the surface tension force σL is called the *Weber number,* named after Moritz Weber, who studied the effects of flow in capillary tubes.

$$W_e = \frac{Inertia\ force}{Surface\ tension\ force} = \frac{\rho v^2 L}{\sigma} \ldots\ldots\ldots\ldots (8.4)$$

  - ✓ It is important because capillary flow is controlled by the force of surface tension within a narrow passageway, provided the Weber number is less than one.

  - ✓ Surface tension force becomes important when studying the flow of thin films of liquid over surfaces, or the flow of small-diameter jets or sprays.

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Important Dimensionless Numbers (cont'd)

- **Mach Number:**

  - ✓ The square root of the ratio of the inertia force $\rho v^2 L^2$ to the force that causes the compressibility of the fluid is called the *Mach number.*

  - ✓ This ratio was developed by Ernst Mach, an Austrian physicist who used it as a reference to study the effects of compressible flow.

  - ✓ Fluid compressibility is measured by its bulk modulus, E_v, which is pressure divided by the volumetric strain, $E_v = p/(\frac{\Delta V}{V})$.

  - ✓ Since the units of E_v are the same as those of pressure, and pressure produces a force $F = pA$, the *force* of compressibility has length dimensions of $F = E_V L^2$.

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Important Dimensionless Numbers (cont'd)

- **Mach Number (cont'd):**

  - ✓ The ratio of the inertia force, $\rho v^2 L^2$, to the force of compressibility then becomes $\rho v^2 / E_V$.

  - ✓ For an ideal gas, $E_V = \rho c^2$, where c is the speed at which a pressure disturbance (sound) will naturally travel within a fluid medium.

  - ✓ So the Mach number becomes,

  - ✓ $$M = \sqrt{\frac{Inertia\ force}{Compressibility\ force}} = \frac{v}{c} \ldots\ldots\ldots\ldots (8.5)$$

  - ✓ If M is greater than one, as in the case of supersonic flow, then inertial effects will dominate, and the fluid velocity will be greater than the velocity c of the propagation of any pressure disturbance.

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Reference

1. Prof. Hossain's Research articles on Dimensional Analysis – You can google as "M. Enamul Hossain + Dimensional Analysis" to find the documents.

2. Fluid Mechanics, R.C. Hibbeler, Pearson Education Canada.

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Question?

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Good Luck

and

Thanks for Being with my Couse

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